RESEARCH ARTICLE

Fuzzy wg**- Connectedness in Fuzzy Topological Spaces

Satyamurthy V Parvatkar[1], Kiran Potadar[2], Basayya B Mathad[3]
[1]Department of Mathematics, Jain College of Engineering and Research,
Belagavi, Karnataka, India
[2]Department of Mathematics, Angadi Institute of Technology and Management,
Belagavi, Karnataka, India
[3]Department of Mathematics, SGBIT, Belagavi, Karnataka, India

ABSTRACT: The aim of this paper is to introduce new class of Fuzzy sets, namely wg**-closed fuzzy set for Fuzzy topological spaces. This new class is properly lies between the class of closed Fuzzy set and the class of wg-closed fuzzy set, we also introduce application of wg**-closed fuzzy sets, the concept of fuzzy wg**-connectedness in Fuzzy topological spaces and their properties are obtained.

KEYWORDS: fwg**-Connected, fwg**-open, fwg**-continuous

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INTRODUCTION

In 1965, Zadeh [1] introduced of the concept of 'fuzzy subset' and in the year 1968, Chang [2] introduced the structure of fuzzy topology as an application of fuzzy sets to general topology. Subsequently many researchers like, Wong [3], Warren [4], Lowen [5], Azad [6], Mukherjee [7], Balasubramanian and Sundaram [8] and many others have contributed to the development of fuzzy topological spaces. The theory of general topological spaces is a special case of the theory of FTS.

The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by Chang [6] and Warren [4] are included. Fuzzy topological spaces and some basic concepts and results on fuzzy topological spaces from the works of Chang [2] ,Warren [4], and Wong[3] are presented. Levine [9] introduced generalized closed sets (g-closed sets) in general topology as a generalization of closed sets. Many researchers have worked on this and related problems both in general and fuzzy topology. In this paper we introduced new class of fuzzy sets namely, fuzzy wg**-connected spaces and some of their properties are also obtained. Fuzzy Topology is a new area of Mathematics that has emerged consequent upon the introduction of fuzzy subsets by Zadeh [1] in the year1968.

The concept of a fuzzy set, various operations on fuzzy subsets such as union, intersection and complementation of fuzzy subsets are discussed. Lists of related properties are included. The image and the inverse image of fuzzy subsets under Zadeh's functions and their properties proved by Chang [2] and Warren [4] are included. FTS and some basic concepts and results on FTS are reported from the works of Chang [2], Warren [4], and Wong [3]. Some basic preliminaries are included. Levine [9] introduced generalized closed sets (g-closed sets) in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. For example, it was proved that a g - closed subset of a compact space is compact. Many researchers have reported on this and related problems both in general and fuzzy topology.

Applying the concepts of g-closed sets in general topological spaces, several results in general topology were improved by introducing and studying g-closed maps by Malghan [10] in 1982 and g-continuous maps by Balachandran, Sundaram and Maki[11] in 1991. Further g* - closed sets, g*-continuous maps were introduced and studied by Veerakumar [12] in the year 2000 for general topological spaces.

The concepts of g*-closed sets and $T^*_{1/2}$, $^*T_{1/2}$ - spaces were introduced and studied by Veerakumar [12] in the year 2000 for general topological spaces. Also g - continuous maps were introduced and studied. The concepts of g* - closed fuzzy sets and $T^*_{1/2}$, $^*T_{1/2}$ - fuzzy spaces

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**Corresponding Author: satyaparvatkar@gmai.com*

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were introduced and studied by Jayasheela Reddy [13] in the year 2002 and Devi and Muthtamil Selvan [14] in the year 2004, for FTS. Also g - continuous fuzzy maps were introduced and studied.

The aim of this work is to introduce a new class of fuzzy sets, namely weakly g**- closed fuzzy sets for fuzzy topological spaces. This class is obtained by weakly generalizing ĝ - open fuzzy sets via fuzzy α-closure.

2. PRELIMINARIES

Throughout this paper (X,T), (Y, σ)& (Z, η) or (simply $X, Y, \& Z$) represents non-empty fuzzy topological spaces on which no separation axiom is assumed unless explicitly stated. For a subset A of a space (X, T).cl(A), int(A) & C(A) denotes the closure, interior and the compliment of A respectively.

Definition 2.01: A fuzzy set A of afts (X,T) is called:

1) A semi-open fuzzy set, if A≤ cl(int(A)) and a semi-closed fuzzy set, if int(cl(A))≤0 [7]
2) A pre-open fuzzy set, if A ≤ int(cl(A))f and a pre-closed fuzzy set, if cl(int(A))≤A [10]
3) Aα-open fuzzy set, if A ≤ int(cl(int(A))) and α a -closed fuzzy set, if cl(int(cl(A)))≤A [15]
4) The semi closure (respectively pre-closure, α-closure) of a fuzzy set A in a fts (X,T)is the intersection of all semi closed (respectively pre closed fuzzy set, α-closed fuzzy set) fuzzy sets containing A and is denoted by scl(A) (respectively pcl(A), αcl(A)).

Definition 2.02: A fuzzy set A of afts (X, T) is called:

1) A generalized closed (g-closed) fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T)) [16].
2) A generalized pre-closed (gp-closed) fuzzy set, if pcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) [17].
3) A α-generalized closed (αg-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) [17, 18 &19].
4) A generalized α-closed (gα-closed) fuzzy set, if αcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X,T) [17, 18 &19]
5) A generalized semi pre closed (gsp-closed) fuzzy set, if spcl(A) ≤ U, whenever A ≤ U and U is open fuzzy set in (X, T) [17,18 &19].
6) A g*-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T) [17].
7) A g**-closed fuzzy set, if cl(A) ≤ U, whenever A ≤ U and U is g-open fuzzy set in (X,T) [17,18].
8) A wg**closed fuzzy set, if cl(int(A) ≤ U, whenever A ≤ U and U is g*-open fuzzy set in (X,T) .[20]

Complement of g-closed fuzzy (respectively gp-closed fuzzy set, αg-closed fuzzy set, gα-closed fuzzy set, gsp-closed fuzzy set, g*-closed fuzzy set , g**-closed fuzzy set and wg**-closed set) sets are called g-open (respectively gp-open fuzzy set, αg-open fuzzy set, gα-open fuzzy set, gsp-open fuzzy set, g*-open fuzzy set, g^{**}-open fuzzy and wg**open fuzzy set) sets.

Definition 2.03: Let X and Y be two fuzzy topological Spaces, A function f: X→Y is called:

1) A fuzzy continuous (f-continuous) if f^{-1} (A) is closed fuzzy set in X, for every closed fuzzy set A of Y.[8]
2) A fuzzy α-continuous (fα-continuous) if f^{-1} (A)is α-closed fuzzy set in X, for every closed fuzzy set A of Y.[23]
3) A fuzzy generalized-continuous (fg-continuous) if f^{-1} (A) is g-closed fuzzy set in X, for every closed fuzzy set A of Y.[8]
4) A fuzzy generalized α-continuous (fgα-continuous) if f^{-1} (A) is gα-closed fuzzy set in X, for every closed fuzzy set A of Y.[15]
5) A fuzzy α-generalized continuous (fαg-continuous) if f^{-1} (A) is αg-closed fuzzy set in X, for every closed fuzzy set A of Y.[15]
6) A fuzzy g*-continuous (fg*-continuous) if f^{-1} (A) is g*-closed fuzzy set in X, for every closed fuzzy set A of Y.[15]
7) A fuzzy wg^{*}-continuous (fwg*-continuous) if f^{-1} (A) is fwg*-closed fuzzy set in X, for every closed fuzzy set A of Y.[19]
8) A fuzzy wg^{**}-continuous (fwg**-continuous) if f^{-1} (A) is fwg**-closed fuzzy set in X, for every closed fuzzy set A of Y.[20]
9) A fuzzy wg^{**}-irresolute (fwg**-irresolute) if f^{-1} (A) is fwg**-closed fuzzy set in X, for every closed fuzzy set A of Y.[20]

Definition 2.04: A map f: X→ Y is called:

1) fuzzy –open (f-open) iff f(V) is open –fuzzy set in Y for every open fuzzy set in X [4]
2) fuzzy g-open (fg -open) iff f(V) is g-open –fuzzy set in Y for every open fuzzy set in X [9]
3) fuzzy wg*-open (fwg*-open) iff f(V) is fwg*-open –fuzzy set in Y for every open fuzzy set in X [19]

4) fuzzy wg**-open (fwg**-open) iff f(V) is fwg**-open –fuzzy set in Y for every open fuzzy set in X [20]

APPLICATIONS

In mathematics, topology is concerned with the properties of geometric object that are preserved under continuous deformation such as stretching, twisting, crumpling and bending, but not tearing or gluing. Fuzzy Topology is used in many branches of mathematics, such as differential equations, dynamical systems, knot theory and Riemann surfaces in complex analysis. It is also used in string theory in physics and for describing the space time structure of universe. The study of topology is relevant to many the concept namely, algebra, analyses, category theory, quantum mechanics, dynamics, geometry, mathematical modeling, mathematics of communication, operation research and many more. Topology may also refer to the collection of open sets used to define a topological space.

3. fwg**-CONNECTEDNESS IN FTS

Definition 3.01: A fts X is said to be fwg**-Connected (briefly fwg**-connected) if X is not a disjoint union of two non empty fwg**-open sets. A subset A of X is fwg**-connected if it is fwg**-connected as a subspace.

Theorem 3.02: For fts X, the following are equivalent

- X is fwg**-connected
- The only subset of X which are both fwg**-open and X is fwg**-closed are the empty set ø and X.
- Each X is fwg**-continuous map of X into a discrete space Y with at least two points is a constant map.

Proof: i) ⇒ii) Let U be a X is fwg**-open and X is fwg**-closed subset of fts X. Then X-U is both X is fwg**-open and X is fwg**-closed. Since X is the disjoint union of the X is fwg**-open sets U and X-U. One of these must be empty, that is U=Ø or U=X.

ii) ⇒i) Suppose that X=AUB where A and B are disjoint non empty fwg**-open subsets of fts X, Then A is both fwg**-open and fwg**-closed. By hypothesis, A=ø or X. Therefore X is fwg**-connected.

ii) ⇒iii) Let f: X→Y be a fwg**-continuous map. Then X is connected by fwg**-open and fwg**-closed covering $\{f^{-1}(y):y\in Y\}$.By assumption , $f^{-1}(y)$ =Ø or X for each y∈Y. If $f^{-1}(y)$ =Ø for all y∈Y then f fails to be a map. Then there exist only one point y∈Ysuch that $f^{-1}(y)$ ≠Ø and hence $f^{-1}(y)$ = X which shows that f is constant map.

iii) ⇒ii) Let U be both fwg**-open and fwg**-closed in fts X. Suppose U≠Ø. Let f: X→Y be a fwg**-continuous map, defined by f(U) = {y} and f(X-U)= {w}for some disjoint points y and w in Y. By assumption, f is constant. Therefore y=w and U=X.

Theorem 3.03: Every fwg**-connected space is connected.

Proof: Let X be a fwg**-connected space. If possible let X be not connected. Then X can be represented as X=AUB, where A and B are disjoint nonempty αg-open sets in X. By hypotheses cl(int(A))≤U and A≤U, since A is g-closed, but cl(int(A))≤cl(A)≤U. Therefore cl(int(A))≤U. Hence A is fwg**-fuzzy closed set. With this hypotheses it proves that every αg-open set is fwg**-open. Thus X=AUB where Aand B are disjoint non empty fwg**-open set in X. This contradicts that X is not connected .Therefore X is connected.

Theorem 3.04: If f: X→Y is fwg**-continuous surjection and X is fwg**-connected. Then Y is connected.

Proof: Let us assume that Y is not connected. Let Y=AUB where A and B are disjoint non empty fwg**-open sets in Y. Since f is fwg**-continuous one-one and onto, X=$f^{-1}(A)$U$f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty fwg**-open sets in fts (X,T).This contradicts that X is fwg**-connected. Therefore Y is connected.

Theorem 3.05: If f: X→Y is a fwg**-gc-irresolute surjection and X is fwg**-connected. Then Y is connected.

Proof: Let us assume that Y is fwg**-connected. Then Y=AUB, where A and B are non empty fwg**-open sets in Y. Since f is fwg**-gc-irresolute, where $f^{-1}(A)$ and $f^{-1}(B)$ are fwg**-open sets in fts X. Since f is onto, $f^{-1}(A)$ and f^{-1} (B) are non empty. Now X=$f^{-1}(Y)$=$f^{-1}(AUB)$= $f^{-1}(A)$ U $f^{-1}(B)$. Then X is a union of disjoint, nonempty and fwg**-open sets in X. This contradicts that X is fwg**-connected. Therefore Y f wg**- connected.

Theorem 3.06: If f: X→Y is strongly fwg**-continuous map from a connected space X onto a fts T. Then Y fwg**- connected.

Proof: Let us assume Y be not fwg**-connected. Then Y can be represented in the form Y=AUB, where A and B are disjoint non empty fwg**-open set in Y. Sine f is strongly fwg**-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are αg-open sets in X. Also X= $f^{-1}(Y)=f^{-1}(AUB)= f^{-1}(A)Uf^{-1}(B)$.This contradicts that X is connected. Therefore Y is connected.

Theorem 3.07: If f: X→Y is completely fwg**-continuous map from a connected space X onto a fts T. Then Y is fwg**-connected.

Proof: Let us assume Y be not fwg**-connected. Then Y can be represented in the form Y=AUB, where A and B are disjoint non empty fwg**-open set in Y. Sine f is fwg**-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are αg-regular open sets in X. Also X= $f^{-1}(Y)$ = f(AUB)= $f^{-1}(A)$ U $f^{-1}(B)$.This contradicts that X is connected. Therefore Y is connected.

CONCLUSION

We observe that every fuzzy weakly g**-connected space is connected but not conversely and the image of a fuzzy weakly g**-connected space under the fuzzy weakly g**-continuous surjection is connected.

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